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D – 2507

Reg. No. : .....

Name : .....

Fifth Semester B.Sc. Degree Examination, December 2017
First Degree Programme Under CBCSS
MATHEMATICS
Core Course : VI
MM1543 : Differential Equations and their Applications
(2013 Admn.)

Time : 3 Hours

Max. Marks : 80

SECTION – I

Answer **all** questions from this Section. **Each** question carries **1** mark.

1. Find the integral curve of the equation $xy' = 1$ passing through $(1, 0)$.
2. Find the orthogonal trajectories of the curves $xy = c$.
3. Write the order of the differential equation $\frac{d^3y}{dx^3} + 2\left(\frac{d^2y}{dx^2}\right)^2 - \frac{dy}{dx} + y = 0$.
4. Find the general solution of the differential equation $y' = e^x - x$.
5. Find an integrating factor for the differential equation $ydx + (x^2y - x)dy = 0$.
6. If $y_1 = x$ is a solution of $x^2y'' + xy - y = 0$. Find a second linearly independent solution.
7. Write the auxiliary equation of $y'' + y' - 6y = 0$.
8. Find the general solution of $y'' - 8y = 0$.
9. Find the Wronskian of $y_1 = x$ and $y_2 = x^2$.
10. Find the Laplace transform of $f(x) = 5e^{-2x} - 3 \sin 4x$, $x \geq 0$.

(10×1=10 Marks)

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SECTION – II

Answer **any eight** questions from this Section. **Each** question carries **2** marks.

11. If the air resistance acting on a falling body of mass m exerts a retarding force pro-portional to the square of the velocity, $\frac{dv}{dt} = g - v^2$. Where $c = k/m$ $v(0) = 0$, find v as a function of t. What is the terminal velocity in this case ?

12. Evaluate $\int_0^{\infty} \frac{\sin x}{x} dx$.

13. Determine N(x, y) such that the equation $(x^3 + xy^2)dx + N(x, y)dy = 0$ is exact.

14. Prove that two solutions of the homogeneous equation $y'' + P(x)y' + Q(x)y = 0$ on an interval [a, b] are linearly dependent if they have a common zero x_0 in the interval.

15. Show that $L(x \cos ax) = \frac{p^2 - a^2}{(p^2 + a^2)^2}$.

16. Transform the equation $y'' + 2y' + 2y = 0$ into a system of first order equations.

17. Solve the differential equation $\frac{dy}{dx} + \frac{1}{2}y = \frac{1}{2}e^{x/3}$.

18. Show that $y = c_1 \sin x + c_2 \cos x$ is the general solutio of $y'' + y = 0$ on any interval and find the particular solution for which $y(0) = 2$ and $y'(0) = 3$.

19. Solve the Euler equation $x^2y'' + xy' + y = 0$.

20. Using Picard's method solve $y' = y$, $y(0) = 1$.

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21. Find L(u(x – a)) where a is a positive number and u(x) is the unit step function.

22. Solve the initial value problem $y' = y^2$, $y(0) = 1$ and determine the interval in which the solution exist.

(8×2=16 Marks)

SECTION – III

Answer **any six** questions from this unit. **Each** question carries **4** marks.

23. Find an integrating factor for the equation $(3xy + y^2) + (x^2 + xy)\,y' = 0$ and then solve the equation.

24. Solve the system $\frac{dx}{dt} = x + y$ and $\frac{dy}{dt} = 4x - 2y$.

25. Show that $\int_0^\infty e^{-s^2} ds = \frac{\sqrt{\pi}}{2}$.

26. A rabbit starts at the origin and runs up the y-axis with speed a. At the same time a dog running with speed b, starts at the point (c, 0) and pursues the rabbit. What is the path of the dog ?

27. Find the general solution of $y'' - 2y' - 3y = 3e^{2x}$.

28. Find a particular solution of $y'' + y = \text{cosec } x$.

29. Show explicitly that $L(x^{-1})$ doesn't exist.

30. Using Laplace Transform solve $y'' + 4y = 4x$ that satisfies the initial conditions $y(0) = 1$ and $y'(0) = 5$.

31. Show that $L\left[x^{-\frac{1}{2}}\right] = \sqrt{\frac{\pi}{2}}$.

(6×4=24 Marks)

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SECTION – IV

Answer **any two** questions from this unit. **Each** question carries **15** marks.

32. Find the general solution of $y'' = 3y' - 4y = 3e^{2x} + 2 \sin x - 8e^{x} \cos 2x$.

33. Find the solution of the initial value problem $y^{(4)} - y = 0$, $y(0) = 0$, $y'(0) = 1$, $y''(0) = 0$, $y'''(0) = 0$.

34. Solve Bessel's equation of order zero $xy'' + y' + xy = 0$, $y(0) = 1$.

35. a) Solve the integral equation $y(x) = e^{x}\left[1 + \int_0^x e^{-t} y(t)dt\right]$.

    b) State the convolution theorem and use it to find the inverse Laplace transform of $H(p) = \dfrac{a}{p^2(p^2 + a^2)}$.

(2×15=30 Marks)

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